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APPLICATION OF THREE-DIMENSIONAL BÉZIER PATCHES IN GRID GENERATION *

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SUMMARY

Bézier and B-spline patches are popular tools in surface modeling. With these methods, a surface is represented by the tensor product of univariate approximations. The extension of this concept to three-dimensions is obvious and can be applied to the problem of grid generation. This report will demonstrate how three-dimensional patches can be used in solid modeling and in the generation of grids. Examples will be given demonstrating the ability to generated three-dimensional grids directly from a wire frame without having to first set up the boundary surfaces. Many geometric grid properties can be imposed by the proper choice of the control net.

INTRODUCTION

Free-form modeling of surfaces has been the main objective in the field of computer aided geometric design. Rational Bézier curves and surfaces have been used extensively for surface modeling. With this method, as well as most other methods of curve and surface modeling, the geometric object is defined using a set of points and a set of basis functions. In the case of surfaces a tensor product basis is used to construct the basis functions for generating a surface patch. Precisely how the basis functions are used in the surface generation depends on the desired properties of the surface. In particular, it depends on whether the set of points that is used to describe the object is to be interpolated as a wire frame for building the surface or is to be a control net to describe the general shape of the surface. It is the first method, usually associated with Coons patches, that will be used to develop a solid modeling technique. However, either concept can be generalize to three dimensions. In

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fact, the patches, which probably should be referred to as blocks in the three-dimensional analogue, are actually defined by constructing a control net.

The purpose of this report is to investigate the concept of free-form modeling of surfaces and apply the idea to the field of grid generation. In the following sections, the generalization of rational Bézier curves to three dimensions is developed and used in the construction of three-dimensional grids. The development is analogous to that used to generate surfaces and follows the terminology and notation of Farin [1]. No doubt the extension of one-dimensional basis functions to arbitrary tensor products in any dimension was known to exist from the earliest development of tensor product surfaces, however, the first application of the concept in solid modeling appeared to be in the paper of Saia *et al.* [2]. Another use of three-dimensional basis functions was also note by Farin.

Several examples of three-dimensional grids constructed from wire frame descriptions are presented. Although no detailed analysis of the quality of the generated grid has been done, it has been shown that in most cases the grid is very close to the grid which is constructed using transfinite interpolation.

RATIONAL BÉZIER PATCHES

A three-dimensional solid is given by a wire frame description. The blocks of the wire frame are filled in with patches and a grid is constructed in each patch. Three-dimensional grids are constructed using the tensor products of one-dimensional basis functions. The basis functions for the rational Bézier curve of degree n are given as

$$R_i^n(t) = \frac{w_i B_i^n(t)}{\sum_{i=0}^n w_i B_i^n(t)}$$

where the B_i^n are the Bernstein polynomials defined as

$$B_i^n(t) = \begin{pmatrix} n \\ i \end{pmatrix} t^i(1-t)^{n-i}.$$

The weight parameters w_i can be used to cause grid clustering near points of the wire frame.

A three-dimensional patch can be constructed from a control net. If the basis functions are of degree l, m and n, then the points of the control net can be denoted as a three-dimensional array $\{\mathbf{b}_{i,j,k}\}, i = 0, \cdots, l,\ j = 0, \cdots, m,\ k = 0, \cdots, n$. The points of the patch are defined using the tensor product basis functions and the points of the control net. The points of the patch are described by the parametric equation

$$\mathrm{b}(r,s,t) = \sum_{i=0}^{l}\sum_{j=0}^{m}\sum_{k=0}^{n} \mathrm{b}_{i,j,k} R_i^l(r) R_j^m(s) R_k^n(t)$$

As in the two dimensions, the case of most interest is when $l = m = n = 3$. For it is then that we have enough control points to ensure continuity of grid line slopes between patches or specify any other type of boundary condition for the grid line slopes. Therefore, in the grid construction in the next section, only rational tricubic Bézier basis functions are implemented.

GRID CONSTRUCTION

The grid construction begins with the input of the coordinates of a wire frame which describes the general shape of the three-dimensional region to be gridded. Along with the point coordinates, there is also input information describing the type of grid line continuity or discontinuity that is to be imposed at the wire frame points. This information is used to prescribe the location of the control points in each block of the wire frame. The description of a rational cubic Bézier curve requires four points, including the two end points of the curve. Therefore each block of the wire frame will require a total of 64 control points including the four vertices on the wire frame. These control points are chosen so that the grid lines have the desired continuity or direction property requested by the input information. So as not to generate an excessive amount of input, the directions of the grid lines were computed using differences of coordinates computed on the wire frame. By using various combinations of central and one-sided difference quotients, the grid lines can be constructed so that they have a discontinuous slope or pass smoothly through the points of the wire frame. Except for the boundary points, a control point is needed on each side of the input point in each of the coordinate directions. These control points indicate the direction of the grid line as it enters and leaves the point. The following list indicates the different types of conditions that have been implemented.

1. Weighted central differences used in both directions.

2. Forward difference used on both sides.

3. Backward difference used on both sides.

4. Backward used going toward point, forward used leaving point.

5. Forward or backward differences at a boundary point.

6. Periodic conditions.

7. Rounded corners.

8. Orthogonal grid lines at a symmetry plane.

The weighting referred to in condition 1 was found to give a better value for the slope. The usual central difference can be formulated as the average of a forward and a backward difference. Here a weighted average of a forward and a backward difference is used where the weighting depends on the distance to the two neighboring points on the wire frame.

Several examples are presented to demonstrate the use of the three-dimensional Bézier patches in grid generation. The first example is plotted in Figure 1. The wire frame model describes a simple duct with a varying cross section. The rounded corner option is used to change the duct from a rectangular shape to an oval shape. The plots of the interior grid surfaces show a uniform distribution of grid points. The weights in the basis functions were all set to one for this example. The second example is more interesting. A minimal number of points is used to describe a wing and part of a fuselage. The bulge below the wing is representative of a faired over engine inlet. Weighting of the basis functions has been used to cluster grid points near the wing/fuselage configuration. The symmetry plane was constructed through the center of the fuselage. Grid lines are orthogonal to the symmetry plane. Different control point directions at the wing tips resulted in the corners on the aircraft surface. The wire frame and several gridded surfaces are shown in Figure 2. The last example is another aircraft configuration. The same number of points as in the previous example is used to model a high speed aircraft. Only the basic wing fuselage geometry is modeled. The grid, plotted in Figure 3, demonstrates the capability of dealing with a polar type of axis and a periodic condition. The sparsity in the number of points which describes the geometry is evident in the wire frame.

There are many three-dimensional grid generation methods that can generate a volume grid from a given surface grid. Thus the question might be asked as to how the grids constructed above using the three-dimensional solid modeling technique differs from the interior grid which would be constructed from a set of bounding surface grids. In an attempt to answer this question, the boundary grids from Figure 3 were used to generate interior grids by transfinite interpolation. The grids constructed by the two methods are not identical, but as can be seen in Figure 4, they are nearly the same, and it would be difficult to say which method generates a better grid. Either grid could be improved by applying an elliptic method to reduce skewness. The grid distribution could also be controlled by adding interior interpolation surfaces in the case of the transfinite interpolation, or by adding points to the wire frame representation in the case of the Bézier grid.

In all of these examples, a uniform knot spacing was used in each patch, however, nonuniform rational Bézier functions could be used or one could use rational B-splines. In any case the basic concept is a straightforward generalization of the surface construction procedure.

One point that is of both practical and mathematical interest has not been treated in this report. That is the three-dimensional analogue of the well-known twist problem in surface modeling. The continuity in the coordinate gradients is not sufficient to uniquely determine all of the control points, and the manner in which these points are chosen determines whether the surface has the correct shape near the vertices of the patch. In our computations, it has been assumed that the mixed derivatives vanish, which in surface modeling is known to result in flat regions near the corners. One of several twist models could have been used, but there would still be left the four interior control points of the block to be determined. Therefore, until there is a reasonable three-dimensional twist model available, it was decided to simply neglect the mixed derivative effects.

CONCLUSIONS

The results of this report show that solid modeling techniques using three-dimensional rational Bézier functions can be used to generate grids in a simple one-step procedure. Many different types of configurations and edge and face treatments can be included in the model. The complexity of the geometric model is only limited by the amount of input that is desired. This solid modeling technique is designed for free-form solids where a general shape or design is to be modeled rather than for constructing a solid with precisely defined edges or faces.

REFERENCES

1 Gerald Farin, *Curves and Surfaces for Computer Aided Geometric Design: A Practical Guide, Second Edition*, Academic Press, San Diego, 1990.

2 A. Saia, M. S. Bloor, and A. De Pennington, "Sculptured Solids in a CSG Based Geometric Modelling System", in *The Mathematics of Surfaces II*, R. R. Martin, Ed., Clarendon, Oxford, 1987.

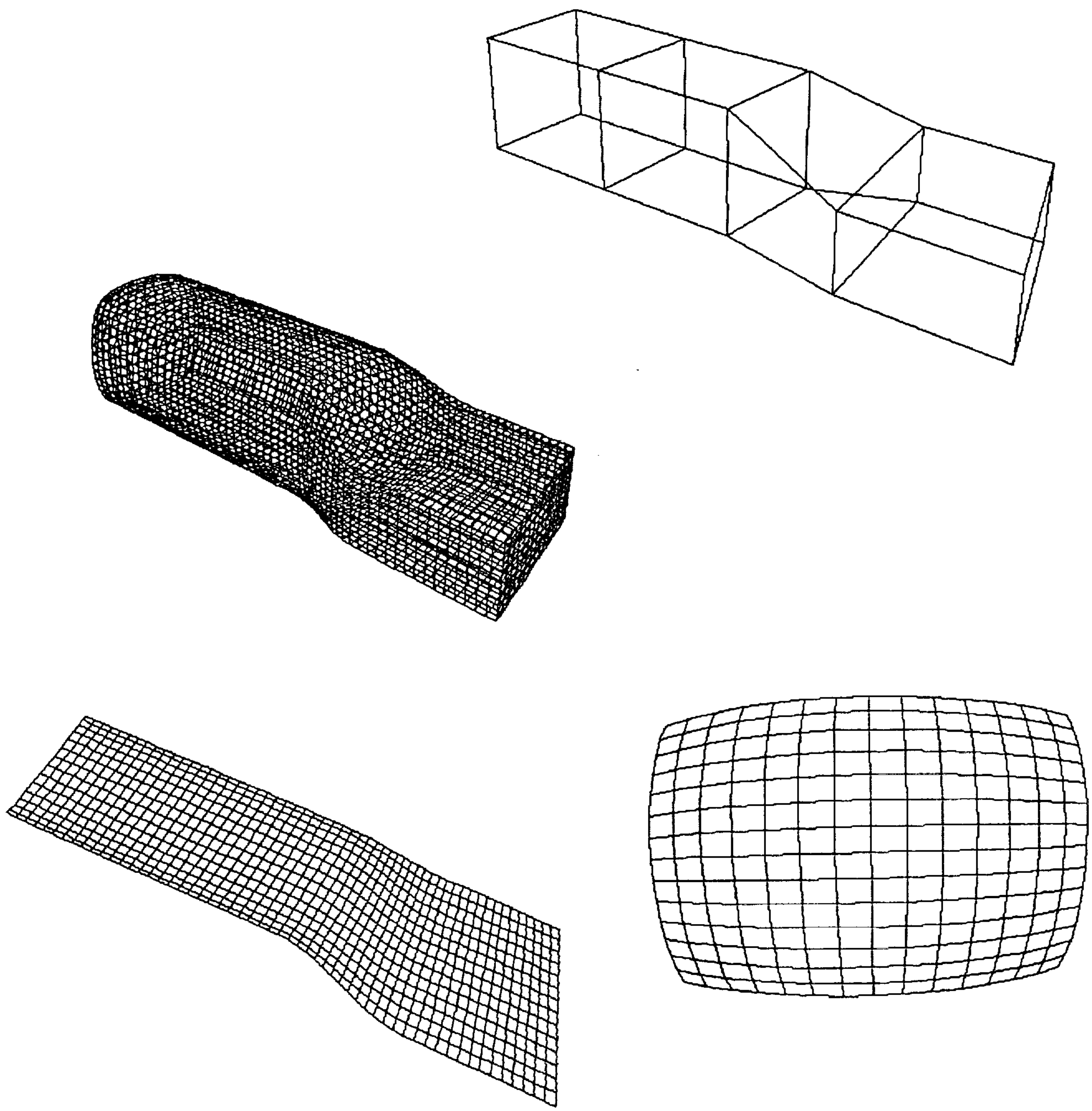

Figure 1. Wire frame and surface grids for a duct with a varying cross section.

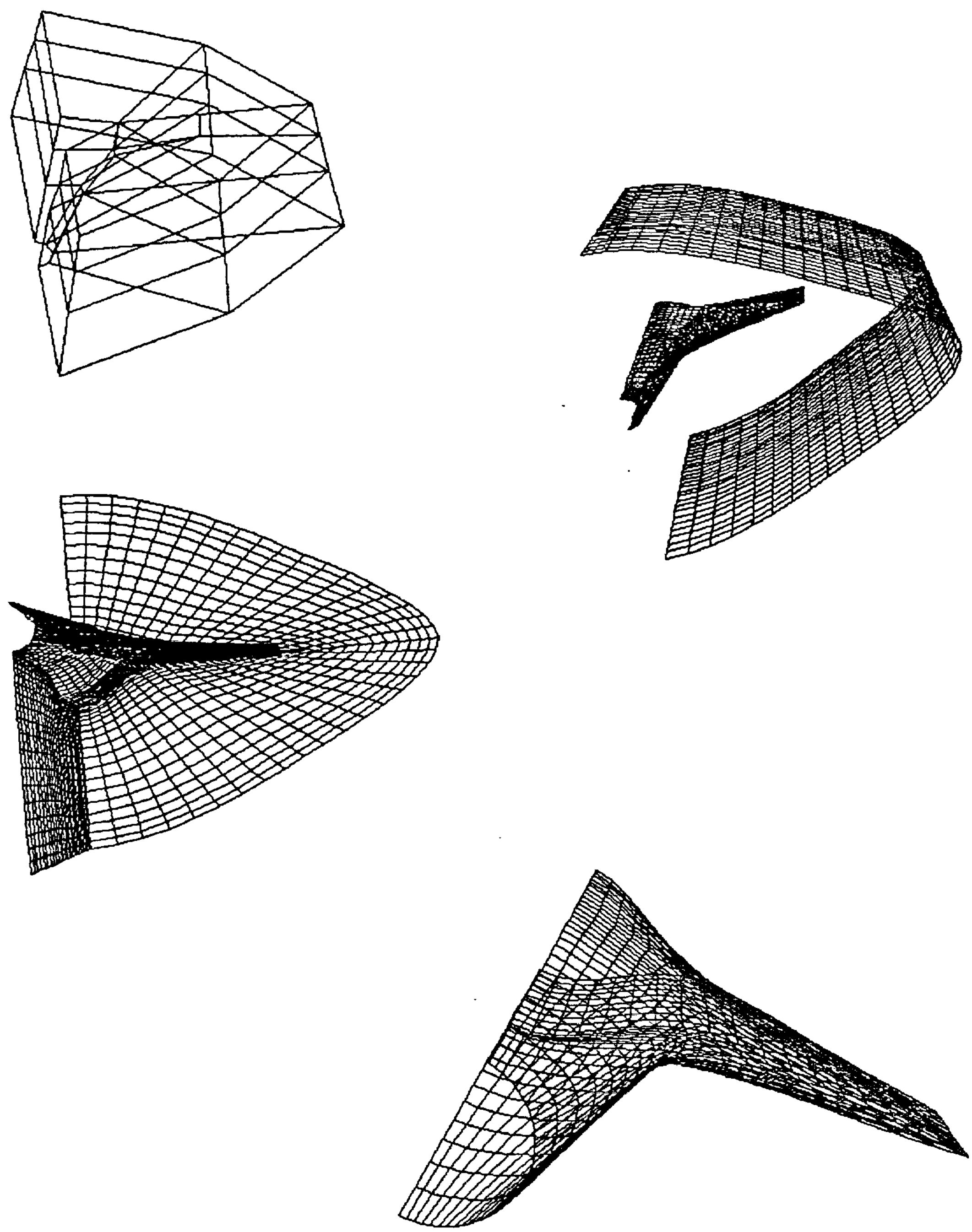

Figure 2. Wire frame and surface grids for a wing and part of a fuselage.

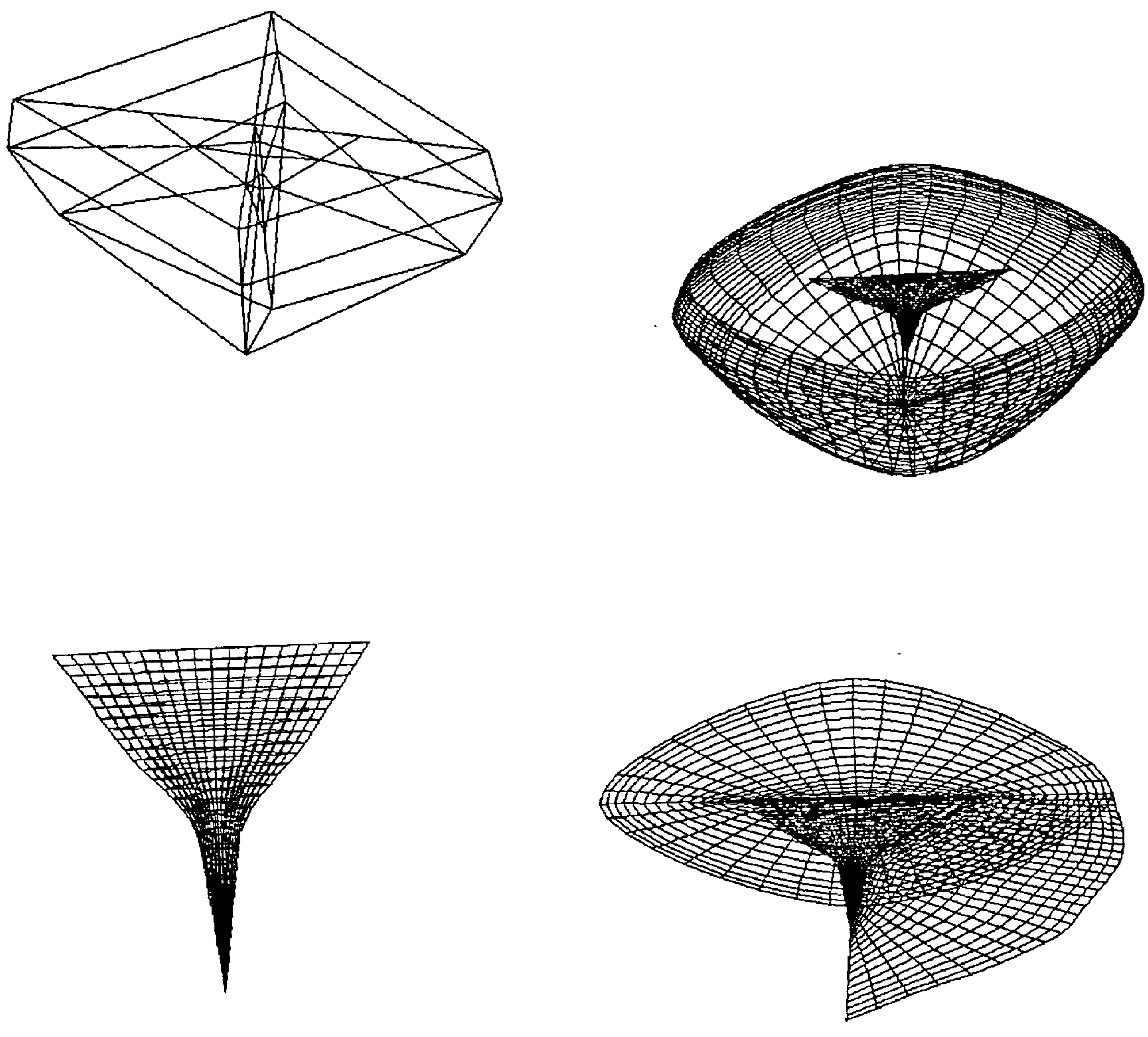

Figure 3. Wire frame and surface grids for a high speed aircraft.

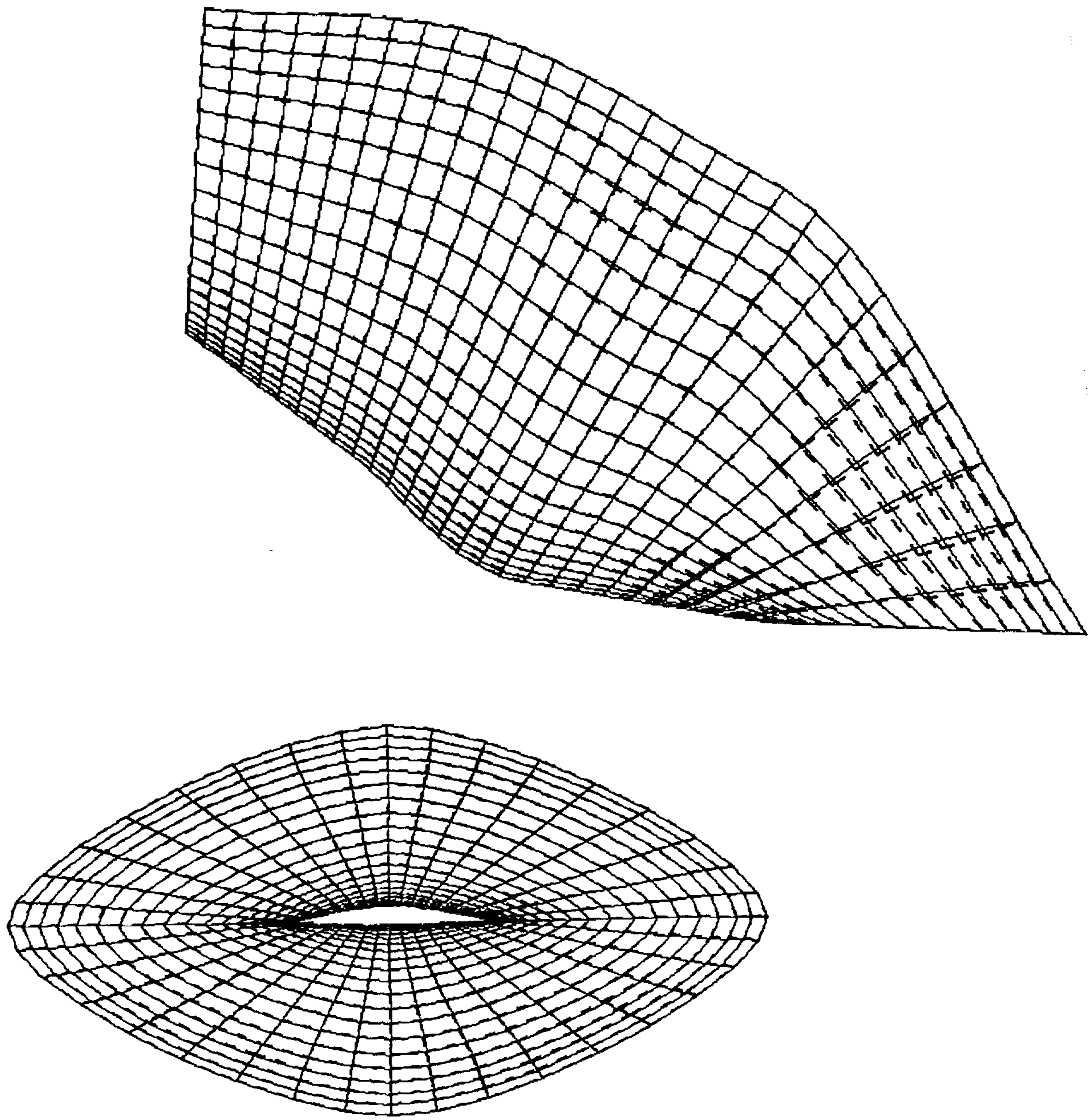

Figure 4. A comparison of 3D Bézier grids (solid lines) and transfinite interpolation grids (dash lines).